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247. Proposed by NORMAN ANNING, Chilliwack, B. C.

To dissect the triangle whose sides are 52, 56, 60 into three Heronian triangles by lines drawn from the vertices to a point within.

The word Heronian is used in the sense of the German Heronische (Wertheim, *Anfangsgründe d. Zahlenlehre*, p. 140) to describe a triangle whose sides and area are integral.

248. Proposed by E. T. BELL, Seattle, Washington.

If $u_{n+2} = 4u_{n+1} - u_n$, with $u_0 = 2$, $u_1 = 4$, prove that the triangle Δ_n, whose sides are $u_n - 1$, u_n, $u_n + 1$, has an integral area; also that all triangles, Δ_n, whose areas are integers, and whose sides are consecutive integers, are given by this process. Hence show that, as n increases, the area of Δ_n approximates $(\sqrt{3}/4)u_n^2$, and find the degree of approximation.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

447. Proposed by ELIJAH SWIFT, University of Vermont.

(*a*) A method is sought of forming an equation such that the first k figures of some root shall be given numbers. For example, form a cubic equation such that one of its roots is 1.918 +.

(*b*) Of all the equations suggested in (*a*), determine that one for which the sum of the absolute values of all the coefficients is least.

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Let the degree of the desired equation be n, the nearest integer to the desired root be x_0, and the root itself be $(x_0 + \Delta x)$.

Choose any $\phi(x)$ of degree $(n - 1)$, and form $y = f(x) = (x - x_0)\phi(x)$.

Compute

$$\Delta y = \Delta x \cdot f'(x_0) + \frac{\overline{\Delta x}^2}{\underline{|2}} f''(x_0) + \cdots + \frac{\overline{\Delta x}^n}{\underline{|n}} \cdot f^{(n)}(x_0)$$

from

$$\Delta y = \Delta x \left[\phi(x_0) + \Delta x \cdot f'(x_0) + \cdots + \frac{\overline{\Delta x}^{n-1}}{\underline{|n-1}} \cdot \phi^{(n-1)}(x_0) \right] \tag{1}$$

or from

$$\Delta y = \Delta x \cdot \phi(x_0 + \Delta x). \tag{2}$$

The more convenient of (1) and (2) should be chosen; usually (1) will be preferable. The computation should be carried to at least as many decimals as are required in the root; if it is important that the root should be greater than $(x_0 + \Delta x)$ rather than less, this is readily managed by slightly increasing or decreasing Δy according as $f'(x_0)$ is > 0 or < 0, with due regard to the fact that $\Delta y \doteq \Delta x \cdot \phi(x_0)$. The desired equation is $f(x) - \Delta y = 0$.

If $n = 3$, and $\phi(x) = \alpha_0 x^2 + \alpha_1 x + \alpha_2$,

$$\Delta y = \Delta x[\alpha_0 x_0^2 + \alpha_1 x_0 + \alpha_2 + (2\alpha_0 x_0 + \alpha_1)\Delta x + \alpha_0 \overline{\Delta x}^2].$$

To form a cubic equation having the root 1.918 +, let $\phi(x) = x^2 - 2x + 7$; then

$$f(x) = x^3 - 4x^2 + 11x - 14,$$

and

$$\Delta y = -.082[4 - 4 + 7 + (4 - 2)(-.082) + (1)(.082)^2] = -.5610 -.$$

Increase Δy by .001 to insure a root > 1.918; then $-\Delta y = .560$, and the desired equation is

$$f(x) - \Delta y \equiv x^3 - 4x^2 + 11x - 13.44 = 0,$$

which has a root 1.918 +.

(*b*) If, in the preceding discussion,

$$\phi(x) = \alpha_0 x^{n-1} + \alpha_1 x^{n-2} + \cdots + \alpha_{n-1},$$

the sum of the absolute values of all the coefficients of $f(x) - \Delta y \equiv (x - x_0)\phi(x) - \Delta y$ is

$$\Sigma = |\alpha_0| + |\alpha_1 - \alpha_0 x_0| + |\alpha_2 - \alpha_1 x_0| + \cdots + |\alpha_{n-1} - \alpha_{n-2} x_0| + |-\alpha_{n-1} x_0 - \Delta x \cdot \phi(x_0 + \Delta x)|. \quad (3)$$

The α's are arbitrary. Let $\alpha_i = \alpha_{i-1} x_0$, so that $\alpha_i = \alpha_0 x_0^i$; then

$$\Sigma = |\alpha_0| + |-\alpha_0 x_0^n - \Delta x \cdot \phi(x_0 + \Delta x)|.$$

The equation $f(x) - \Delta y = 0$ must, of course, become

$$\alpha_0[x^n - (x_0 + \Delta x)^n] = 0, \qquad \text{and} \qquad \Sigma = |\alpha_0|(1 + |x_0 + \Delta x|^n).$$

By choosing α_0 small enough, we can make Σ as small as we please.

This type of equation may not be what is desired, though it serves very well for practice in Horner's Method. In any case, the expression (3) for Σ is a useful guide to a happy medium of simplicity.

448. Proposed by W. D. CAIRNS, Oberlin College.

In the *Washington* (D. C.) *Times*, Mr. W. A. Dayton called attention some time ago to a curious repetition of digits in the decimal value of 1/115. If this decimal, which we print in the form $.0\dot{0}86956521739130\ 4347826\dot{0}$, be divided by two, the result is $.00\dot{4}3478260\ 8695652173913\dot{0}$, the fourteen-digit and eight-digit groups having been thus interchanged. A similar result, as he points out, is obtained if the original decimal value is divided by four. Mr. Dayton asks that this curiosity be explained.

SOLUTION BY H. S. BROWN, Hamilton College, N. Y.

The decimal point was misplaced in the original question. The separation of the decimal into two *groups* of digits has no bearing on the problem.

The following theorem, found in almost any algebra that has a chapter on Theory of Numbers, applies directly: "If $1/p$ be converted into a circulating decimal with $p - 1$ figures in its recurring period, p must be prime, and the recurring period being multiplied by 2, 3, $\cdots$, $(p - 1)$ will reproduce its own digits in the same order." (C. Smith's *Treatise on Algebra*, p. 502.) Proof: If

$$1/p = .\dot{a}_1 a_2 a_3 \cdots \dot{a}_{p-1},$$

we have

$$10 = a_1 p + r_1, \qquad 10 r_1 = a_2 p + r_2, \qquad 10 r_2 = a_3 p + r_3, \quad \cdots.$$

Now p is prime to 10, for otherwise $1/p$ would not be reducible to a pure circulator; hence p is prime to $r_1, r_2, r_3, \cdots$. But as $r_1, r_2, r_3, \cdots$, are all different and there are $p - 1$ of them, they are the numbers 1, 2, 3, $\cdots$, $(p - 1)$, not of course in order. Also from the above, p is prime. Now since $r_1, r_2, r_3, \cdots$ include all numbers 1, 2, 3, $\cdots$ $(p - 1)$, it follows that when k/p is reduced to a decimal the recurring digits will be the same as before, beginning with that one for which the previous remainder was k.

In the above example, we have

$$\frac{1}{115} = \frac{1}{5 \times 23} = \frac{2}{10 \times 23}, \qquad \text{where } k = 2.$$

It is worth noting that if the decimal 1/23 of 22 places is separated into two equal groups and the sum of the digits of the first half added to the sum of the digits of the second half, the result will consist wholly of nines. Thus, $1/23 = .\dot{0}43478260869565217391\dot{3}$.

$$\begin{array}{l l r}
48 & = \text{sum of the first eleven digits,} & 04347826086 \\
\underline{51} & = \text{sum of the second eleven digits,} & \underline{95652173913} \\
99 & & 99999999999
\end{array}$$

This result is general under the condition that the denominator is prime and the number of figures in the recurring period is even. We note also that twos and fives, as factors in the denominator of a fraction, have no effect on the number of recurring figures when the fraction is reduced to a decimal.